Halo effect in the $^{11}Be + {}^{64}Zn$ elastic scattering

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Motivation : experimental data

[A. Di Pietro et al, Phys. Rev. Lett. 105 (2010) 022701]

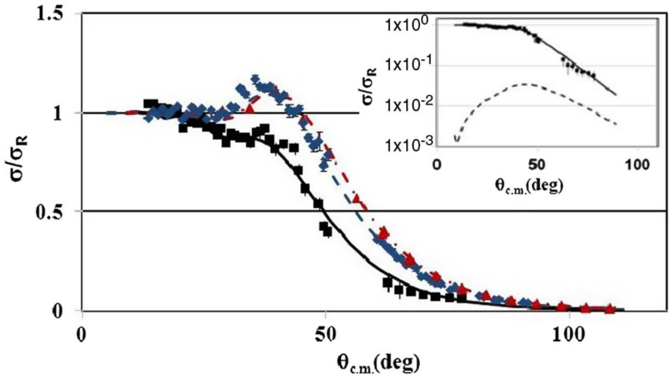


Figure: Elastic scattering angular distributions on ^{64}Zn : ^{9}Be (red triangles), ^{10}Be (blue diamonds) and ^{11}Be (black squares) at $E_{\mathrm{c.m.}} = 24.5$ MeV.

Motivation : importance of continuum

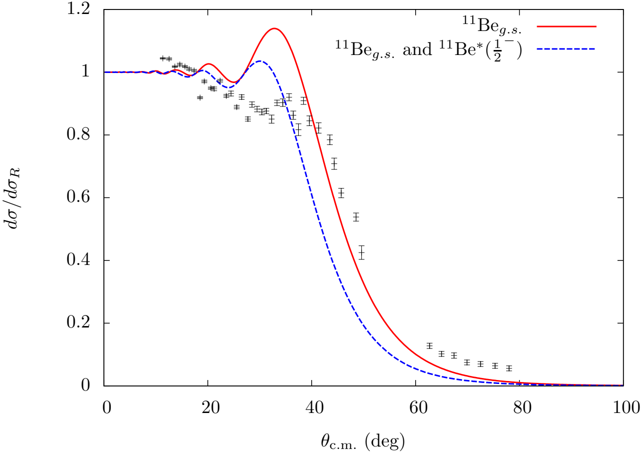


Figure: Elastic cross section : ^{11}Be + ^{64}Zn ($E_{\mathrm{c.m.}} = 24.5$ MeV). Data from [A. Di Pietro et al, Phys. Rev. Lett. 105 (2010) 022701]

Framework [G.H. Rawitscher, Phys. Rev. C 9 (1974) 2210]

CDCC $\equiv$ Coupled Discretized Continuum Channel

Schrödinger's equation : $H\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$

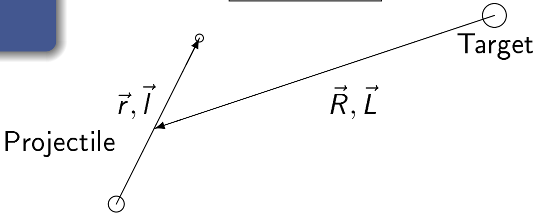
Hamiltonian : $H = H_0 + T_R + V_{tc}\left(\vec{R} + \frac{A_f}{A_p}\vec{r}\right) + V_{tf}\left(\vec{R} - \frac{A_c}{A_p}\vec{r}\right)$

Projectile model

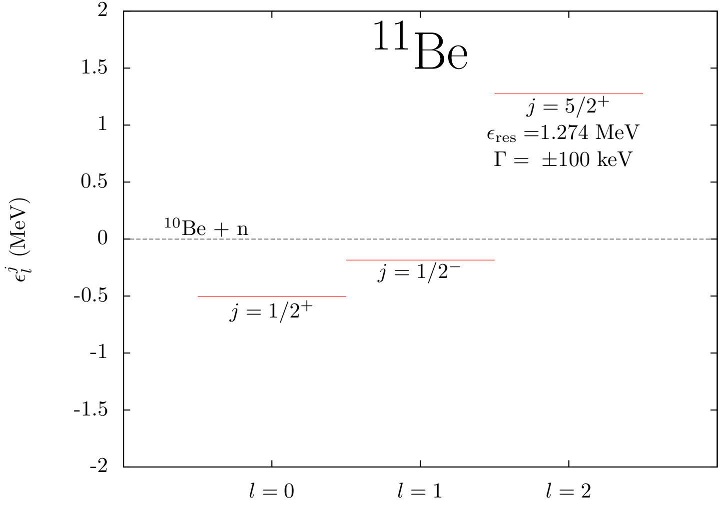
$H_0\Phi_l(\vec{r}) = \epsilon_l\Phi_l(\vec{r})$
$H_0 = T_r + V_{cf}(r)$

Jacobi coordinates

$$\boxed{\vec{J} = \vec{l} + \vec{L}}$$



Projectile model : ^{10}Be (core) + n (fragment)



Principle of coupled channel method

$\Psi(\vec{R},\vec{r})$ is expanded over the projectile wavefunctions

$$\Psi(\vec{R},\vec{r}) = \underbrace{\sum_{il} \Phi_{il}(\vec{r})\chi_{il}(\vec{R})}_{\epsilon<0 \text{ (bound states)}} + \underbrace{\sum_{l} \int \Phi_{\vec{k}l}(\vec{r})\chi_{\vec{k}l}(\vec{R})d\vec{k}}_{\epsilon>0 \text{ (continuum states)}}$$

and introduced into the Schrödinger's equation $H\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$.

Discretized continuum

Reminder : projectile

$$H_0\Phi_{\vec{k}l}(\vec{r}) = \epsilon_l\Phi_{\vec{k}l}(\vec{r})$$
$$H_0 = T_r + V_{cf}(r)$$

l $\equiv$ projectile's angular momentum.

Discretization :

The bin method : definition

$$\Phi_{il}(\vec{r}) := \frac{1}{\sqrt{\Delta_i}}\int_{k_{i-1}}^{k_i} f_{il}(k)\Phi_{\vec{k}l}(\vec{r})dk$$

Breakup states are averaged with a weight function $f_{il}(k)$.

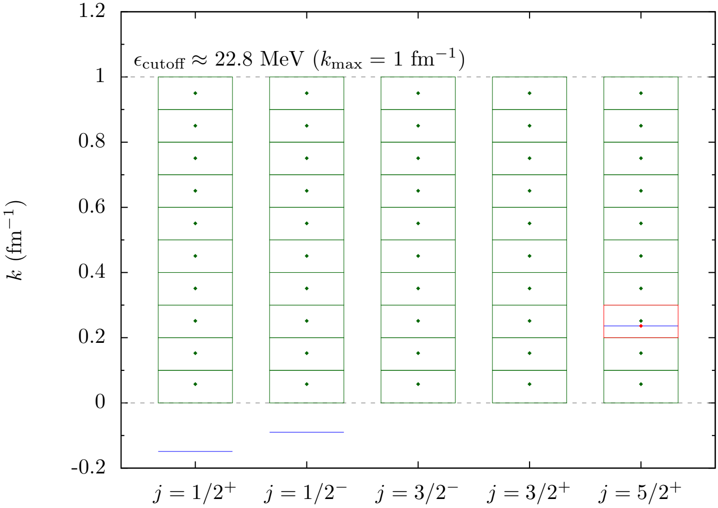
No resonance : $f_{il}(k) = 1$ and $\epsilon_{il} = \frac{\hbar^2\left(k_i^2+k_ik_{i-1}+k_{i-1}^2\right)}{6\mu_{cf}}$

Resonance : $f_{il}(k) = \left|\frac{i\Gamma/2}{\epsilon(k)-\epsilon_{\rm res}+i\Gamma/2}\right|$ and $\epsilon_{il} = \epsilon_{\rm res}$

$$\boxed{\Phi_{\vec{k}l}(\vec{r}) \rightarrow \Phi_{ilm}(\vec{r}) = r^{-1}\phi_{il}(r)Y_l^m(\Omega_r)}$$

Bin method : discretized continuum of ^{10}Be + n

return



Finite expansion (3-body)

$$\begin{aligned}\Psi^{JM\pi}_{\mathrm{CDCC}}(\vec{R},\vec{r}) &= \sum_{il} \Phi_{il}(\vec{r})\chi_{il}(\vec{R}) \\ &= \frac{1}{rR}\sum_{lLi} \underbrace{u^{J\pi}_{ilL}(R)}_{\text{to be determined}} \phi_{il}(r) Y^{JM}_{lL}(\Omega_R,\Omega_r)\end{aligned}$$

$H\Psi^{JM\pi}(\vec{R},\vec{r}) = E\Psi^{JM\pi}(\vec{R},\vec{r})$ replaced by discrete finite system of equations.

$$\left\{-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} + \epsilon_{il} - E\right)\right\} u^{J\pi}_{ilL}(R) + \sum_{i'l'L' \neq ilL} V^{J\pi}_{ilL,i'l'L'} u^{J\pi}_{i'l'L'}(R) = 0$$

Potential matrix elements

The previous discrete finite system was

$$\left\{-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}+\epsilon_{il}-E\right)\right\}u_{ilL}^{J\pi}(R)+\sum_{i'l'L'\neq ilL}V_{ilL,i'l'L'}^{J\pi}u_{i'l'L'}^{J\pi}(R)=0$$

with the potential matrix elements

$$V_{ilL,i'l'L'}^{J\pi}=\left\langle\phi_{il}(r)Y_{lL}^{JM}\left(\Omega_R,\Omega_r\right)\left|V_{tc}+V_{tf}\right|\phi_{i'l'}(r)Y_{l'L'}^{JM}\left(\Omega_R,\Omega_r\right)\right\rangle .$$

With

- i numerical integration over r
- ii analytical integration over (Ω_R,Ω_r)
- iii system of coupled equations which depends on J and π

New tools to solve the system and to find $u_{ilL}^{J\pi}(R)\equiv u_{\gamma}^{J\pi}(R)$:

R-matrix method and Lagrange mesh

Principle : Space divided into two regions

V

internal region : $r \leq a$

external region : $r > a$

0 a R

Nuclear + Coulomb interactions

$$u^{J\pi}_{\gamma,\mathrm{int}}(R) = \sum_{j=1}^{N} c_j \varphi_j(R)$$

$$\varphi_j(R) \equiv N \text{ basis functions}$$

Coulomb interaction only

$$u^{J\pi}_{\gamma,\mathrm{ext}}(R) \rightarrow I_{\gamma'}(KR)\delta_{\gamma\gamma'} - U^{J\pi}_{\gamma\gamma'} O_{\gamma\gamma'}(KR)$$

Asymptotic behavior must be $\frac{Z_p Z_t e^2}{R}$

Principle : Space divided into two regions

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internal region : $r \leq a$

external region : $r > a$

0 $\qquad a \qquad R$

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Asymptotic behavior must be $\frac{Z_p Z_t e^2}{R}$

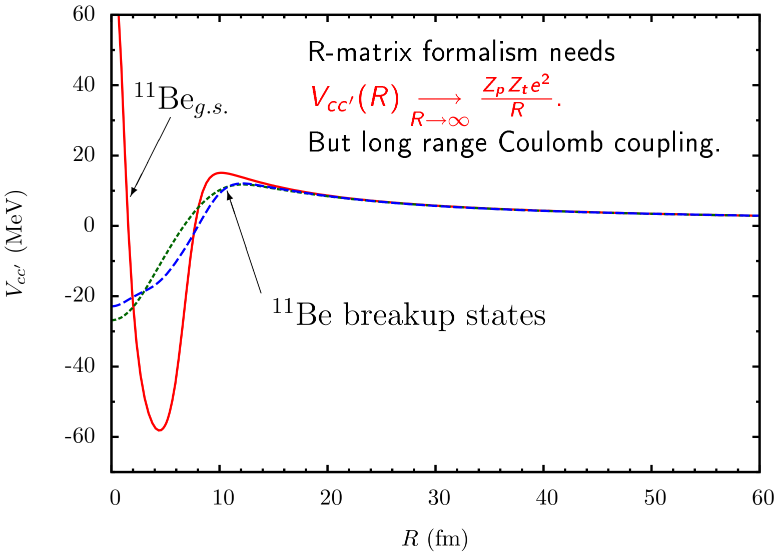
Matching $u^{J\pi}_{\gamma,\mathrm{int}}(R)$ and $u^{J\pi}_{\gamma,\mathrm{ext}}(R)$ at $R = a$ provides

i collision matrix $U^{J\pi}_{\gamma\gamma'}$

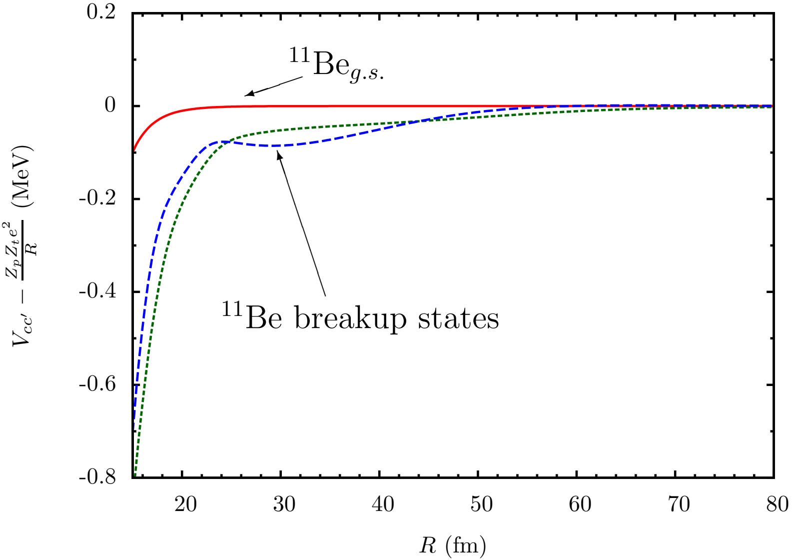
ii coefficients c_j

And the collision matrix $U^{J\pi}_{\gamma\gamma'}$ provides the cross sections (results independent of a).

Potential matrix elements $V^{J\pi}_{ilL,i'l'L'}(R) := V_{cc'}(R)$

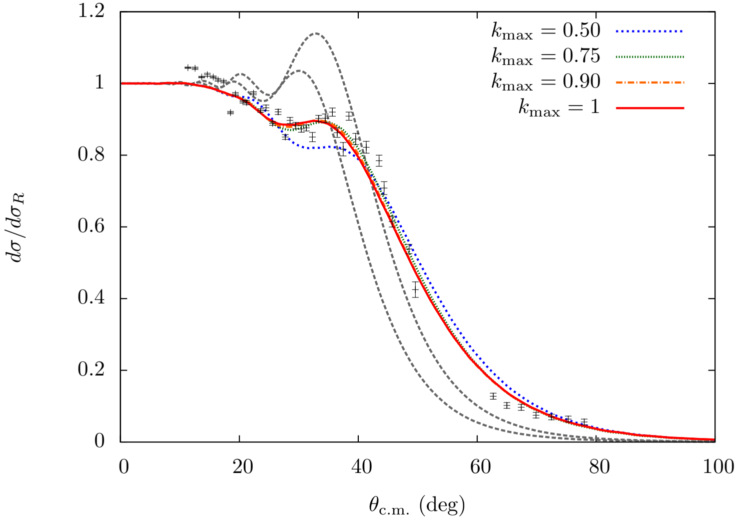


Potential matrix elements : $V_{cc'}(R) - \frac{Z_p Z_t e^2}{R}$

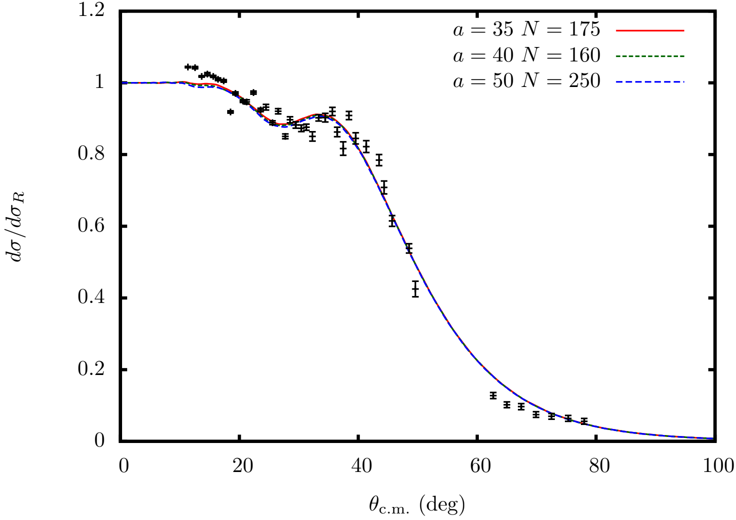


Numerical convergence : cutoff energy

go to bins



Numerical convergence : channel radius



To conclude

- Importance of the breakup channels in the elastic cross section.
- The resonant state has to be properly take into account.
- Numerical convergence is still under examination.

Future work :

- Inelastic and breakup cross sections of ^{11}Be.
- Analysis using a near-side/far-side decomposition [P. Capel et al, Phys. Lett. B 693 (2010) 448].

Thank you for your attention.